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Uniformly convergent finite difference schemes for singularly perturbed 1D parabolic reaction–diffusion problems

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ABSTRACT

In this talk we consider the numerical approximation of the solution of 1D parabolic singularly perturbed problems of reaction-diffusion type. With this aim we combine the classical implicit Euler method on a uniform mesh to discretize in time and two classical finite difference schemes to discretize in space, both defined on a priori special meshes, condensing the mesh points in the boundary layer regions [3]. The scheme is a uniformly convergent method of first order in time and second or third order in space for the central difference scheme or a HODIE method respectively.

This analysis of the uniform convergence is made in two steps, following similar ideas to these ones developed previously for parabolic problems (see for example [2]), where some auxiliary problems were introduced. Here the proof of the uniform convergence is based on a new study of the asymptotic behavior of the exact solution of the semidiscrete problems, which are obtained after the time discretization by the Euler method.

The technique is simpler than its preceding one, because it does need the stability of any discrete transition operator and also permit us to eliminate an artificial relation between the two discretization parameters, arising in the theoretical analysis of some previous works. A detailed analysis of this technique for the implicit Euler and central difference schemes can be found in [1]. We give some numerical results corroborating in practice the theoretical results.

References

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