

An optimal time-stepping for a time dependent advection-diffusion problem[†]

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ABSTRACT

The transport of a pollutant in regions of strong tide-induced currents is mainly governed by a time dependent advection-diffusion problem. The nature of the problem may alternate from diffusion dominated to advection dominated throughout a tidal cycle, see, e.g., [2]. Standard numerical methods may not produce acceptable results in a case when a problem is advection dominated [1].

We consider a 3-point, 2-level weighted scheme for a one-dimensional case of such a problem. The accuracy of the scheme is analyzed by means of its modified equivalent partial differential equation and its wave response properties [3]. A von Neumann analysis is applied to assess the stability of the scheme. By combining these stability results with those obtained from the error analysis, we present an algorithm which produces an optimal time-step as the strength of the advective process varies. Theoretical findings are supported by numerical results.

References

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