

## Finite element discretizations of optimal control flow problems with boundary layers<sup>†</sup>

M. Braack<sup>1</sup>, B. Tews<sup>1</sup>

### ABSTRACT

We study the effect of different stabilized finite element methods to distributed control problems governed by singular perturbed Oseen equations. On the one hand, the residual based stabilized finite element method SUPG/PSPG leads to different optimality systems depending on the discretization approach: first discretize the state equation and then formulate the corresponding optimality system or derive first the optimality system on the continuous level and then discretize it. On the other hand, for symmetric stabilization [1] as for instance the local projection stabilization (LPS) both approaches lead for linear problems to the same symmetric optimality system.

In [2], residual based method have been investigated for optimal control problems with scalar convection-diffusion problems. In this talk, we focus on flow problems and numerically investigate the effect of the upper mentioned methods to the Oseen problem with exponential boundary layers. In particular, we refer to the question whether the commutation error in optimal control problems by residual based methods affect the accuracy significantly or not. It turns out that the answer depends on the polynomial degree of the finite element approach. Furthermore, we address the question to which extend the optimization loop itself may benefit from symmetric stabilization.

Finally, we show first results for optimal control with the nonlinear Navier-Stokes system.

### References

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<sup>1</sup>Mathematisches Seminar  
Christian-Albrechts-Universität zu Kiel  
Ludewig-Meyn-Str. 4, D-24098 Kiel, Germany  
[braackmath.uni-kiel.de](mailto:braackmath.uni-kiel.de), [tews@math.uni-kiel.de](mailto:tews@math.uni-kiel.de)