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High Reynolds Channel Flows : Variable Curvature.

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ABSTRACT

We consider a laminar two-dimensional flow of an incompressible Newtonian fluid in a curved channel at high Reynolds number R_e. Previous analysis, using the asymptotic matching method has been done [2, 3] . A different approach termed SCEM, in which one assumes a uniformly valid approximation (UVA) based on generalized expansions is adopted in the present work. This method developed by Cousteix and Mauss [1] leads to an asymptotic reduced model called **Global Interactive Boundary Layer** (**GIBL**) which avoids the complex and subtle process of constructing formally the asymptotic matching, and gives the solution of the flow field in the whole domain.

The geometrical configuration consists of a 2D bend connected to 2 fitting tangent straight channels at its upstream and downstream extremities. For a point M with curvilinear coordinates X and Y, we have $\overrightarrow{dM} = dX(1+KY)\vec{\tau} + dY\vec{n}$, where $\vec{\tau}$ is the unit vector tangent at M_0 to the median line of the channel in such a way that $(\vec{\tau}, \vec{n})$ is direct; $K(X)$ is the algebraic curvature of this line. Let U and V denote the velocity components parallel and perpendicular to this line, then $\overrightarrow{V} = U\,\vec{\tau} + V\,\vec{n}$.

For high Reynolds numbers and for a small curvature, it can be shown that the Navier-Stokes equations reduce to the **GIBL**,

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0; \quad U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P_1}{\partial X} + \frac{1}{R_e}\frac{\partial}{\partial Y}\left[(1+KY)\frac{\partial U}{\partial Y}\right]$$

These equations must be solved with the boundary conditions, $U = V = 0$ for $Y = \pm\frac{1}{2}$ and the pressure is solution of the linearised Euler equations in the core:
$-u_0\frac{\partial V_1}{\partial Y} + V_1\frac{du_0}{dY} = -\frac{\partial(P_1 - P_0)}{\partial X}$ and $u_0\frac{\partial V_1}{\partial X} - Ku_0^2 = -\frac{\partial P_1}{\partial Y}$ where $u_0(Y) = \frac{1}{4} - Y^2$ and $P_0(X) = -\frac{2X}{R_e}$ denote the basic parallel flow in curvilinear coordinates.

To explore the effect of the variable curvature the bend has an elliptical median line. Comparison of the predicted flow field with the case of a constant curvature will be presented, and the validity of the proposed **GIBL** model is confirmed by confronting the obtained results with numerical solutions of complete Navier-Stokes equations. This comparison includes among other parameters, the skin friction coefficient which is a very sensitive measure of the flow field. The upstream and downstream effects will be also considered.

References

[1] J. COUSTEIX AND J. MAUSS,*Asymptotic Analysis and Boundary Layers*. Scientific Computation, vol. XVIII, Springer, Berlin, Heidelberg, 2007.

[2] F.T. SMITH, On the high Reynolds number theory of laminar flows. *IMA J. Appl. Math.* **28** (3), 207–281, (1982).

[3] T.J. PEDLEY, *The Fluid Mechanics of Large Blood Vessels*. Cambridge University Press, London, 1980.

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