International Conference on Boundary and Interior Layers
Zaragoza, July 5th–9th

Local projection stabilization with projection spaces defined on overlapping sets

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ABSTRACT

It is well known that the Galerkin finite element method is not appropriate for solving convection–dominated problems numerically since the discrete solution is typically globally polluted by spurious oscillations. To enhance the stability and accuracy of the Galerkin method, various stabilization approaches have been developed. We concentrate on stabilizations by local projections which represent an attractive alternative to popular residual–based stabilizations. Among their advantages, let us mention that they do not require the computation of second order derivatives, can be easily applied to non–steady problems and do not create undesirable couplings between various components of the solution if they are applied to systems of partial differential equations. Moreover, they are symmetric and hence very convenient for optimization problems since the operations 'discretization' and 'optimization' commute.

Classical local projection stabilizations use projection spaces defined on mutually disjoint sets, which causes that they require (significantly) more degrees of freedom than, e.g., residual–based methods. We remove this drawback by allowing that the sets on which local projection spaces are defined overlap. Moreover, we shall demonstrate that the overlaps increase the stability of the method and its robustness with respect to the choice of the stabilization parameter.

We shall apply the generalized version of the local projection stabilization to a steady scalar convection–diffusion–reaction equation and present a stability and error analysis with respect to the SUPG norm that is stronger than the usual local projection norm. The stabilization term is slightly modified, which leads to an optimal estimate of the consistency error even if the stabilization parameters scale correctly with respect to convection, diffusion and mesh width. The theoretical considerations will be illustrated by numerical results. In particular, a computational comparison of the local projection method and the SUPG method will be presented.

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