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Flux difference schemes for singularly perturbed parabolic equations; approximation of solutions and derivatives

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ABSTRACT

Grid approximation of an initial-boundary value problem is considered for a singularly perturbed parabolic reaction-diffusion equation on a vertical strip. The spatial derivatives in the differential equation, which are written in divergent form, are multiplied by the perturbation parameter ε^2, where ε takes arbitrary values in the open-closed interval $(0, 1]$. Parabolic boundary layers appear in a neighbourhood of the strip boundary for small values of ε.

To construct a special finite difference scheme, piecewise-uniform basic grids are used that condense in the boundary layers [3]; on the base of these grids, flux grids are introduced [1, 2]. Using the integro-interpolational method [1, 2], an ε-uniformly convergent flux finite difference scheme is constructed, which is conservative [1, 2]. The flux scheme couples discrete solutions and fluxes in x_1 and x_2 on the flux grids, moreover, the conservation law [1, 2] holds for the discrete solutions on the basic grids. The discrete solutions converge ε-uniformly at the rate $\mathcal{O}\big(N_1^{-2}\,\ln^2 N_1 + N_2^{-1} + N_0^{-1}\big)$, where N_1+1 and N_2+1 are the number of mesh nodes on the x_1-axis and the minimal number of nodes on the x_2-axis per unit length, respectively, and N_0 is the number of nodes on the t-axis.

Note that normalized derivatives $\varepsilon^k\,(\partial^k/\partial x_1^k)\,u(x,t),\ k = 1, 2$, in the direction across the boundary layer, and the derivatives $(\partial^k/\partial x_2^k)\,u(x,t),\ k = 1, 2$, along the boundary layer, are ε-uniformly bounded. Relevant discrete derivatives converge ε-uniformly at the same convergence rate as the discrete solutions.

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References

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