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Robust finite difference schemes approximating the solution and derivatives with improved accuracy

G. Shishkin[1]

ABSTRACT

Robust numerical methods, i.e., methods for which solution errors are independent of the perturbation parameter [2], are well developed for singularly perturbed boundary value problems but these methods have the low convergence rate. So, for parabolic reaction-diffusion equations with sufficiently smooth solutions, order of ε-uniform convergence for difference schemes on special meshes does not exceed 2 in x and 1 in t. A technique developing for the improvement of accuracy for regular problems turns out to be inapplicable for constructing robust numerical methods of improved accuracy.

At present, robust numerical methods with improved accuracy are rather intensively developed (see, e.g., [2] and the bibliography therein) but approximation of derivatives to solutions in these methods is not considered. Note that derivatives (normalized) obtained for improved solutions of robust difference schemes do not inherit the accuracy of the improved solutions. Therefore, the development of robust difference schemes that approximate both the solution and its derivatives with improved ε-uniform accuracy is actual. Convergence of discrete solutions is considered in the maximum norm [1, 2] since the solutions have large gradients.

In the present research, a robust finite difference scheme is constructed and studied for a model Dirichlet problem for a singularly perturbed ordinary differential equation; the solution of this scheme converges ε-uniformly (in the maximum norm) with improved convergency rate, namely, with the third order of accuracy. The use of piecewise-uniform grids, which condense in the boundary layer, guarantees the robustness of the scheme constructed; to increase accuracy of the scheme, the Richardson technique is applied. This difference scheme allows us also to approximate the normalized derivatives $\varepsilon\,(d/dx)\,u(x)$ and $\varepsilon^{\,2}(d^2/dx^2)\,u(x)$, which are ε-uniformly bounded, with the same convergence rate as the discrete solution. The applicability of the approach developed is discussed for problems to partial differential equations.

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References

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[1]Institute of Mathematics and Mechanics, Russian Academy of Sciences
S. Kovalevskaya Str., 16, GSP-384, 620219 Ekaterinburg, Russia
`email_shishkin@imm.uran.ru`