

About a stabilized finite element method for the poro-elasticity problem and its resolution by multigrid

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ABSTRACT

In this communication we consider a poroelasticity problem formulated as a system of partial differential equations for the unknowns displacement $\mathbf{u} = (u, v)$ and pressure p . We consider the case of an homogeneous, isotropic and incompressible medium in a domain Ω . The governing equations are given by

$$\begin{aligned} -\mu\Delta\mathbf{u} - (\lambda + \mu)\text{grad div } \mathbf{u} + \text{grad } p &= \mathbf{g}(\mathbf{x}, t), \\ \frac{\partial}{\partial t}(\text{div } \mathbf{u}) - \frac{\kappa}{\eta}\Delta p &= f(\mathbf{x}, t), \quad \mathbf{x} \in \Omega, \quad 0 < t \leq T. \end{aligned}$$

where λ and μ are the Lamé coefficients, κ is the permeability of the porous medium and η the viscosity of the fluid. It is typical for this problems that, due to a lack of compatibility between the boundary conditions and the initial condition, a transient boundary layer can appear. It is well-known that non-physical oscillations of the numerical solution can happen when non stabilized discretizations are considered. A stabilized $P_1 - P_1$ finite element scheme for the poroelasticity equations, based on the perturbation of the flow equation, was proposed in [1]. It allows to use very simple approximation spaces for both displacements and pressure, obtaining solutions without oscillations, independently of the chosen discretization parameters. An important aspect is the efficient resolution of the corresponding linear system arising from the discretization of the problem. We present here an efficient geometric multigrid method for this stabilized finite element discretization of the poroelasticity problem on semi-structured grids, based on box-type smoothers.

References

- [1] G. AGUILAR, F.J. GASPAR, F.J. LISBONA AND C. RODRIGO, Numerical stabilization of Biot's consolidation model by perturbation on the flow equation. *Int. J. Num. Meth. Eng.* **75**, 1282-1300, (2008).

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