

## The error function in the study of singularly perturbed convection-diffusion problems with discontinuous boundary data

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### ABSTRACT

We consider the model convection-diffusion problem  $-\varepsilon\Delta U + \vec{v} \cdot \vec{\nabla}U = 0$  in a set  $\Omega$  in  $\mathbf{R}^2$  or  $\mathbf{R}^3$ , with  $\varepsilon > 0$  and  $\vec{v}$  a constant vector. Besides the small perturbation parameter  $\varepsilon$ , other sources of singular behaviour for the solution of singular perturbation problems are the possible discontinuities of the boundary data. We consider for this problem Dirichlet boundary data piecewise constant:  $U|_{\partial\Omega} = 0$  or 1 with jump discontinuities at some points in  $\partial\Omega$ . We analyze this problem in different two and three-dimensional unbounded and bounded domains  $\Omega$  with discontinuous boundary data at  $\partial\Omega$ . For all these problems, we find that the solution in the singular limit  $\varepsilon \rightarrow 0^+$  and away from the discontinuity points of the boundary data can be approximated uniformly in the form  $U = U_0(1 + \mathcal{O}(\sqrt{\varepsilon}))$ , where  $U_0$  is an error function or a combination of error functions. We show that the error function provides an excellent approximation and reproduces accurately the effect of the discontinuities on the behaviour of the solution at the boundary and interior layers.

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