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Uniform convergence of monotone Newton-like iterates for semilinear singularly perturbed problems

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ABSTRACT

In the study of numerical methods for nonlinear singularly perturbed problems, the two major points have to be developed: i) constructing parameter uniform difference schemes; ii) obtaining reliable and efficient computing methods for computing nonlinear discrete problems. A fruitful method for the treatment of these nonlinear discrete problems is the monotone method (known as the method of lower and upper solutions). The monotone method leads to iterative methods which converge globally and solve only linear discrete systems at each iterative step which is of great importance in practice.

This paper deals with numerical solving semilinear singularly perturbed problems of the elliptic type

$$Lu + f(x, y, u) = 0, \quad (x, y) \in \omega,$$

$$u(x, y) = g(x, y), \quad (x, y) \in \partial\omega,$$

where ω is a bounded domain and $\partial\omega$ is the boundary of ω. We consider the linear differential operator L in the form $Lu = -\mu^2(u_{xx} + u_{yy})$ or $Lu = -\varepsilon(u_{xx} + u_{yy}) + b_1 u_x + b_2 u_y$, where μ and ε are small parameters.

For solving nonlinear discrete approximations, we construct a monotone iterative method with the quadratic convergence of iterations, based on the accelerated monotone iteration process. Since the initial iteration in this monotone method can be constructed from the equation without any knowledge of the solution, this approach simplifies considerably the search for the initial guess as is often required in the Newton's method and possesses the quadratic convergence of iterations. Uniform convergence in the perturbation parameter of the monotone iterative method is investigated and convergent rate is estimated. Numerical experiments complement the theoretical results.

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